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**Upper limit power for self-guided  
propagation of intense lasers in  
plasma**

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# Outline

- Motivation: why do we need self-guided propagation of intense lasers
- Relativistic self-focusing threshold
- Ponderomotive force effect: upper limit power and lower limit plasma density
- Anti-plasma channel for self-guided propagation?
- Summary

# Self-guiding of intense lasers through a long distance is crucial for many applications



Remote sensing devices using lasers, ***Lidar***: for **L**ight **d**etection and **r**anging.

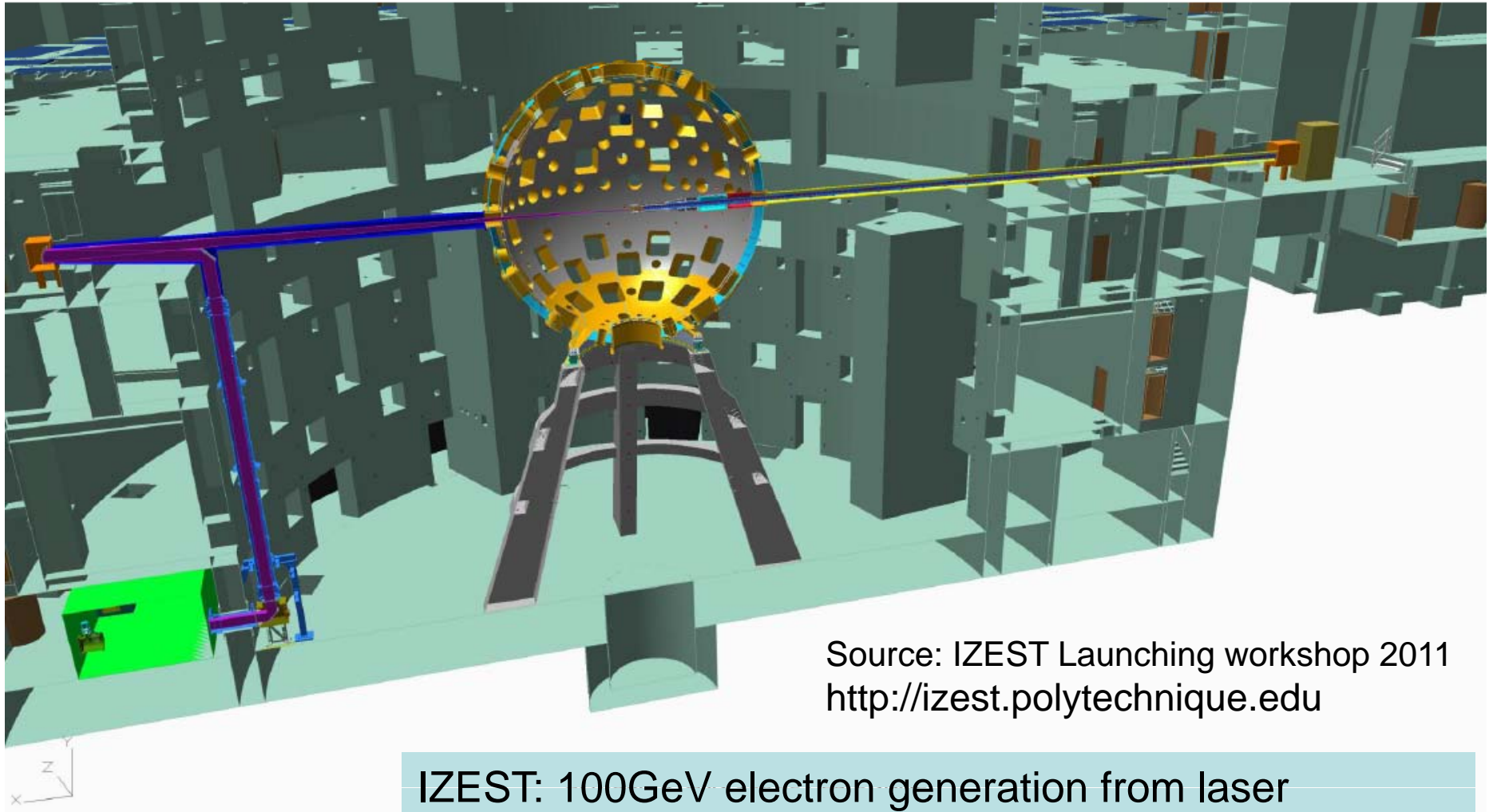
Source: Teramobile



Lightning control using lasers

Source: <http://sparkingdawn.com>

# Self-guiding of lasers is crucial for many applications



Source: IZEST Launching workshop 2011  
<http://izest.polytechnique.edu>

IZEST: 100GeV electron generation from laser  
wakefield on PETAL  
PETAL: 3.5 kJ, 1053 nm, 0.5 ~10 ps

# Self-focusing of lasers in plasma

$$\eta = \sqrt{1 - \frac{4\pi e^2 n_e}{\gamma m_e \omega_0^2}}$$

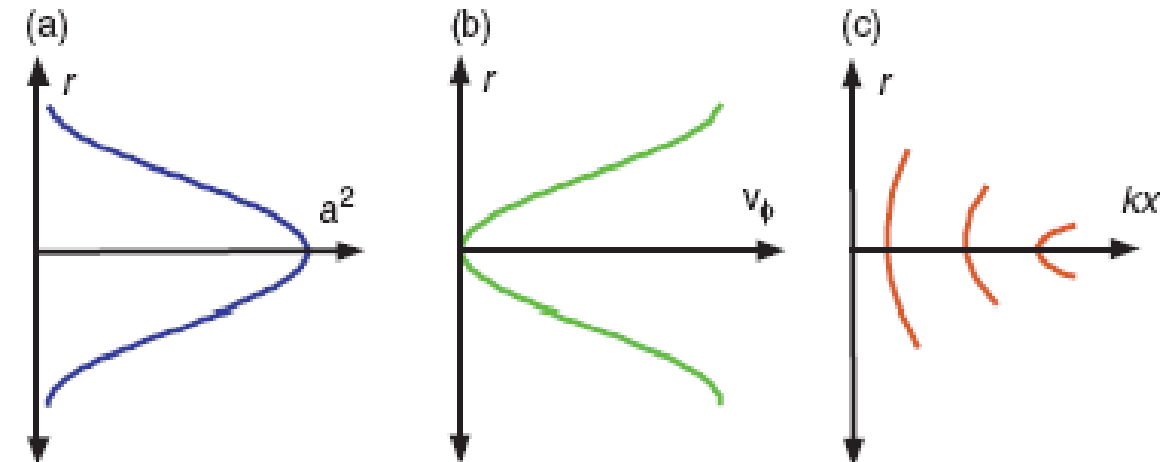
$$v_\phi = c/\eta$$


Diagram (a) shows a blue curve representing the beam radius  $r$  versus intensity  $a^2$ . The curve is symmetric about the horizontal axis and has a minimum at the center. Diagram (b) shows a green curve representing the beam radius  $r$  versus phase velocity  $v_\phi$ . The curve is symmetric about the horizontal axis and has a minimum at the center. Diagram (c) shows an orange curve representing the beam radius  $r$  versus normalized distance  $kx$ . The curve is symmetric about the horizontal axis and has a minimum at the center.

$$\text{Diffraction spreading} \left[ \nabla_{\perp}^2 + 2ik \frac{\partial}{\partial z} \right] a = - \frac{\omega_p^2}{c^2} \frac{|a|^2}{4} a \quad \text{Relativistic self-focusing}$$

- Usually both relativistic effect (**change of electron mass  $m_e$** ) and transverse ponderomotive force (**change of electron density  $n_e$** ) may lead to laser self-guiding in plasma.
- When  **$P_0 > P_c = 17(n_c/n_e)GW$** , relativistic self-focusing can overcome defocusing, according theory in the weakly relativistic case. <sup>6</sup>

# Self-guiding in laser wakefield acceleration

Matched beam spot size in the bubble regime or blowout regime:

$$k_p R \simeq k_p w_0 = 2\sqrt{a_0}.$$

$$a_0 \simeq 2(P/P_c)^{1/3}$$

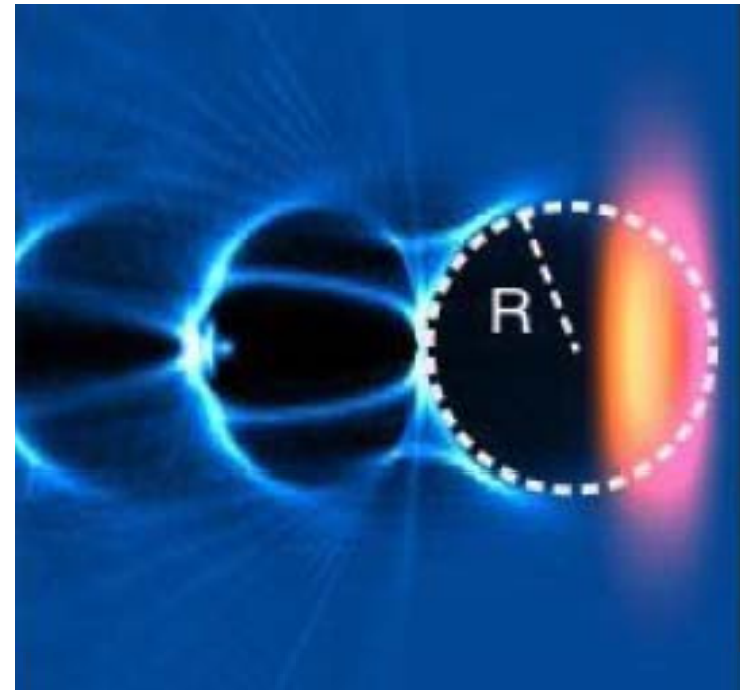
$R$ : the blowout radius

$w_0$ : the laser spot radius

$k_p$ : the plasma wave vector

$a_0$ : the normalized laser peak amplitude

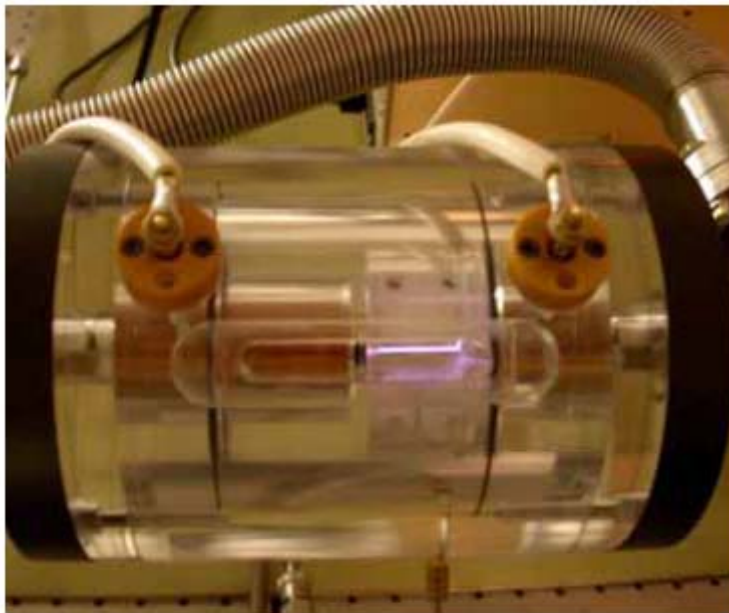
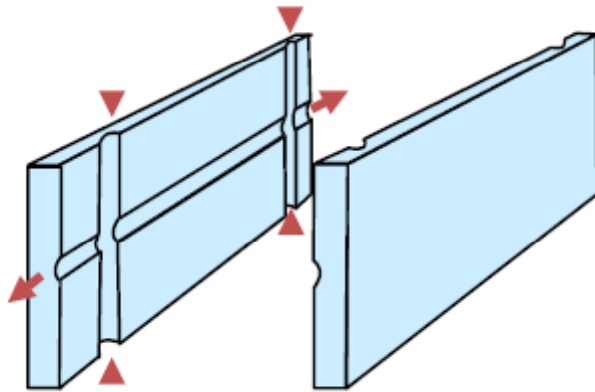
$P_c = 17(n_e/n_c)$  (GW)



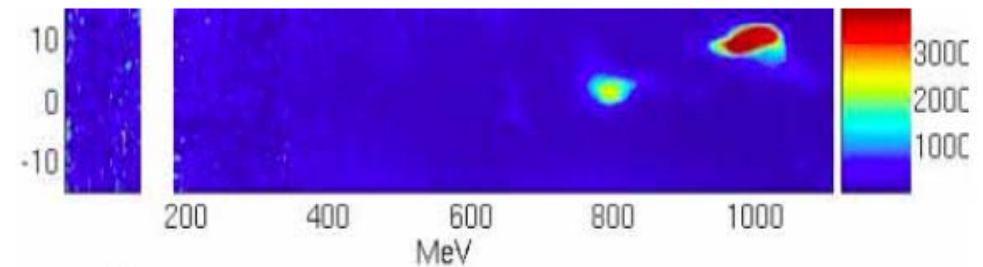
W. LU et al. *Phys. Rev. ST Accel. Beams* **10**, 061301 (2007) 7



# Plasma channels are often adopted



GeV beams from gas-fill  
capillary at LBNL-Oxford



Laser: 40TW 37fs ( $a=1.4$ )  
Capillary: 312 $\mu$ m diam. , 33mm length  
Plasma  $n_e$ :  $4.3 \times 10^{18} \text{ cm}^{-3}$

W.P. Leemans et al., Nature Physics 2,  
696 (2006);

D. J. Spence et al. Phys. Rev. E 63  
015401(R) (2001)



# Does the self-focusing criterion holds for PW lasers?

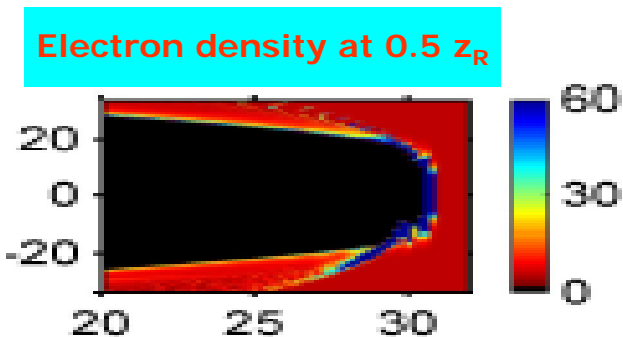
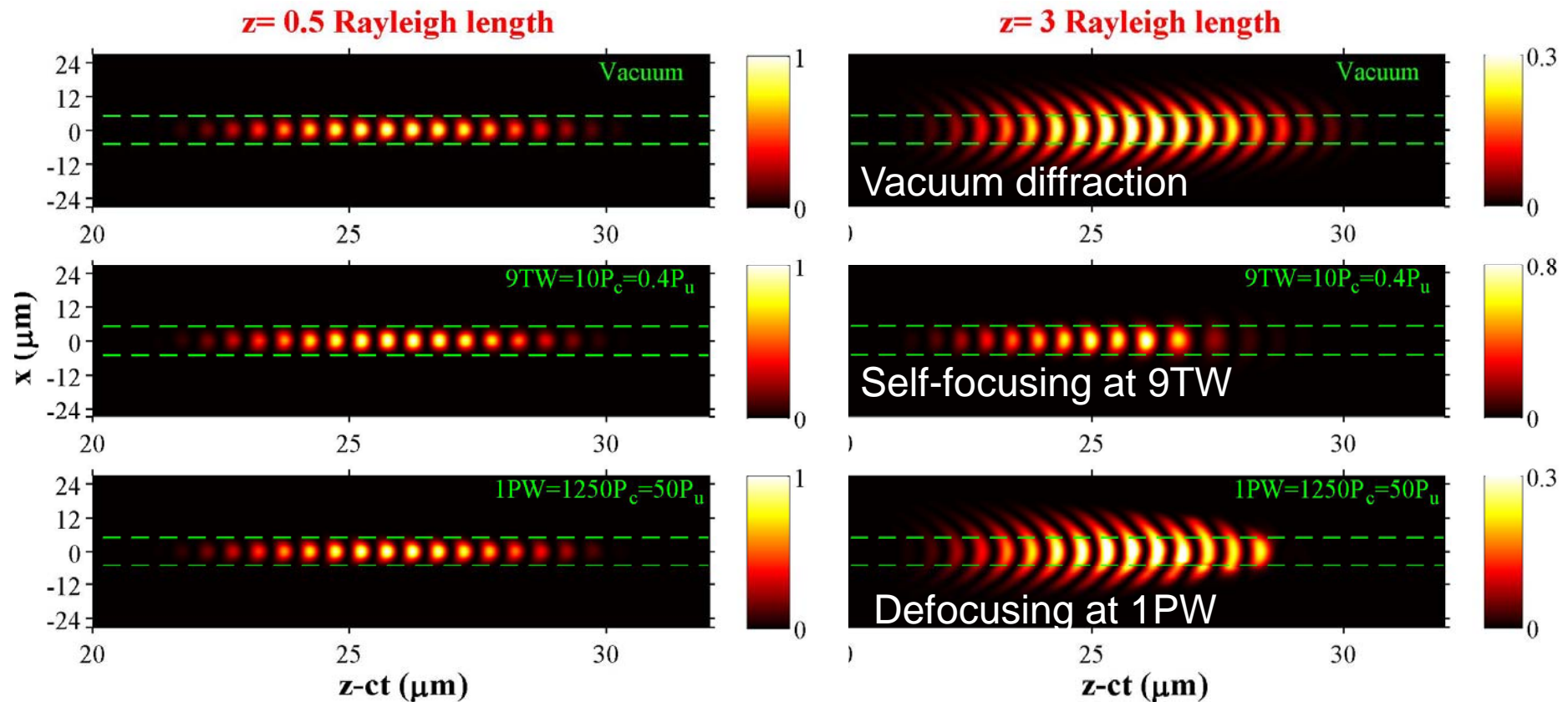
## ■ Current situation ( $\leq 100$ TW for most experiments)

- (a)  $P_c = 17(n_e/n_c)$  (GW) is broadly adopted in LWFA designs and experiments, and confirmed in simulations;
- (b) General view: ponderomotive force helps focusing, which produces channel-like plasma density distributions. Often one generates such preformed channels for laser guiding

## ■ What about for PW lasers or tens or hundreds of $P_c$ for future experiments? Our results indicate that

- (a) Ponderomotive defocusing may occur
- (b)  $P > P_c$  is not enough for self-focusing and there is upper limit power  $P_u$  for self-focusing, i.e.,  $P_c < P < P_u$

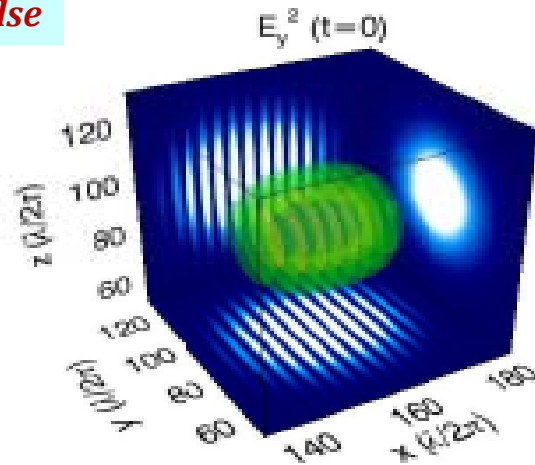
# Ponderomotive defocusing of PW lasers (far above $P_c$ ) (2D PIC simulations)



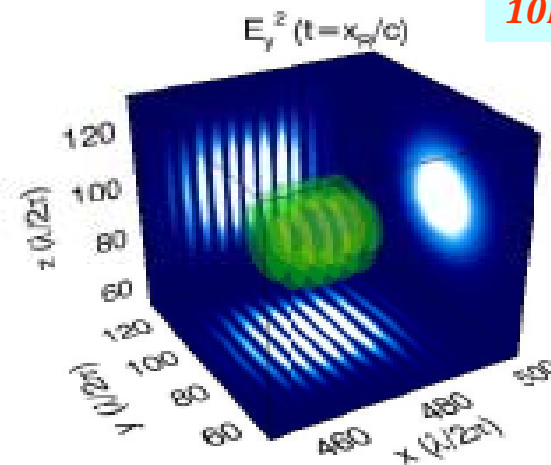
Defocusing is found even for PW lasers, even though self-focusing is found at 9TW.

# Similar results are found in 3D PIC simulations

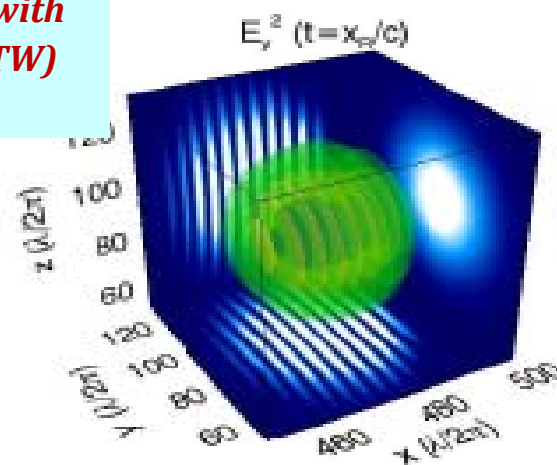
*Initial pulse*



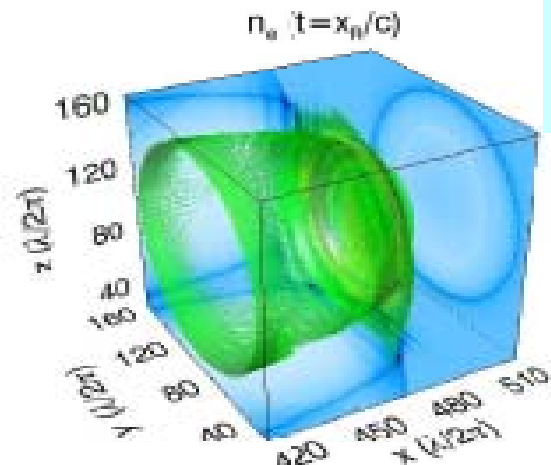
*Self-focusing with  
10Pc (9.2TW) at  $z_R$*



*Defocusing with  
320Pc (294TW)  
at  $z_R$*



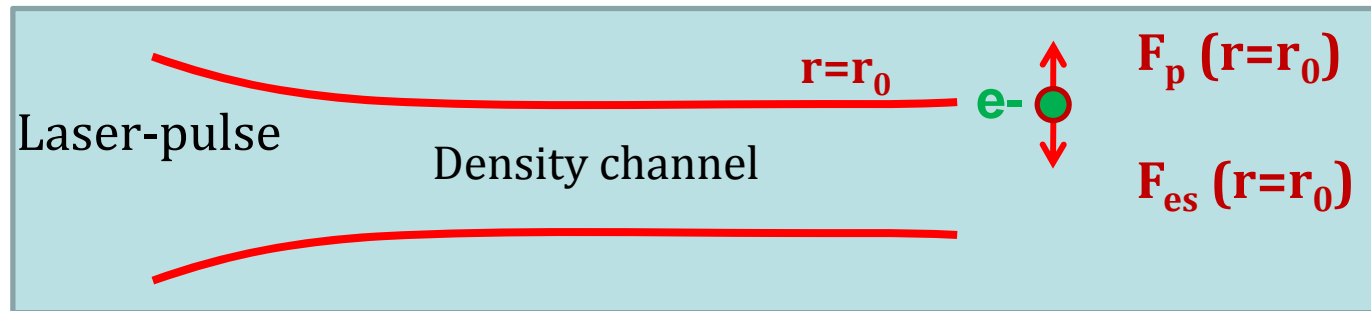
*Electron density  
with 320Pc (294TW)  
at  $z_R$*



## Similar results are found in 3D PIC simulations

Both 2D and 3D PIC simulations indicate there is an **upper-limit power** in addition to the **lower-limit critical power**  $P_c = 17(n_e/n_c)$  (GW) for self-focusing

## Upper-limit power $P_u$ for self-focusing or power threshold for ponderomotive defocusing



Self-guided propagation requires

$$\mathbf{F}_p(r = r_0) + \mathbf{F}_{es}(r = r_0) = 0$$

**$F_p$ : transverse ponderomotive force**

**$F_{es}$ : transverse Coulomb force**

**$r_0$ : laser beam radius,**

**Laser:**  $e\mathbf{A}/mc^2 = \hat{e}_y a_0 \sin(\pi\xi/\tau_0) \sin(\omega\xi) \exp(-r^2/r_0^2)$

$$\vec{F}_p = -mc^2 \nabla_{\perp} \gamma, \quad \vec{F}_{es} = -\hat{e}_r 2\pi n_0 e^2$$

$$\gamma \approx 1 + a^2 / 2 \quad \text{or} \quad (1 + a^2)^{1/2}$$

## Upper-limit power $P_u$ for self-focusing or power threshold for ponderomotive defocusing

$$F_p(r = r_0) + F_{es}(r = r_0) = 0$$

$$\implies \begin{aligned} P_u^{3D} &= \frac{n_{e0} r_0^4}{n_c \lambda^4} \times 3.1 \text{ TW} && \text{3D geometry} \\ P_u^{2D} &= 2 P_u^{3D} && \text{2D slab geometry} \end{aligned}$$

**Note that:**

$$\begin{aligned} P_c^{3D} &= 17(n_c / n_{e0}) \text{ GW} && \text{3D geometry} \\ P_c^{2D} &\rightarrow P_c^{3D} / \sqrt{2} && \text{2D slab geometry} \end{aligned}$$

$P_c$ : due to the relativistic effect  
 $P_u$ : due to the ponderomotive force

## Lower-limit density $n_L$ for self-focusing

- For laser self-guiding, it is required that the laser power  $P$  satisfies:  $P_c < P < P_u$

$$P_c < P < P_u \quad \Rightarrow \quad n_{e0} > 0.074 n_c (\lambda^2 / r_0^2)$$

- A lower-limit density for self-guiding

$$n_L = \frac{\lambda^2}{r_0^2} \times 0.074 n_c$$

3D geometry

$$n_L = \frac{\lambda^2}{r_0^2} \times 0.044 n_c$$

2D slab geometry

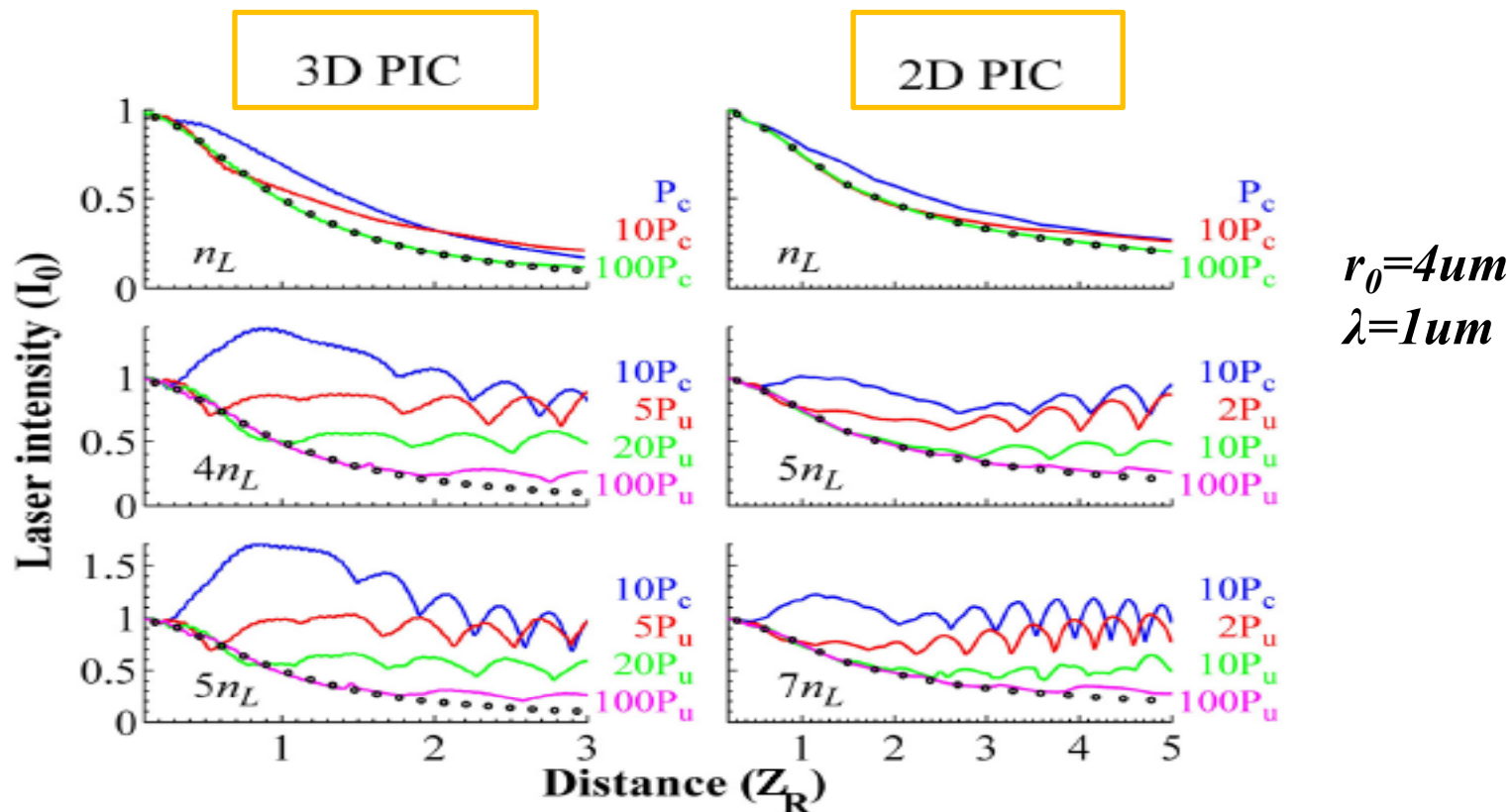
- The relation of  $P_u$  and  $P_c$  in terms of  $n_0$  and  $n_L$

$$P_u = \left( \frac{n_{e0}}{n_L} \right)^2 P_c$$

2D or 3D geometry

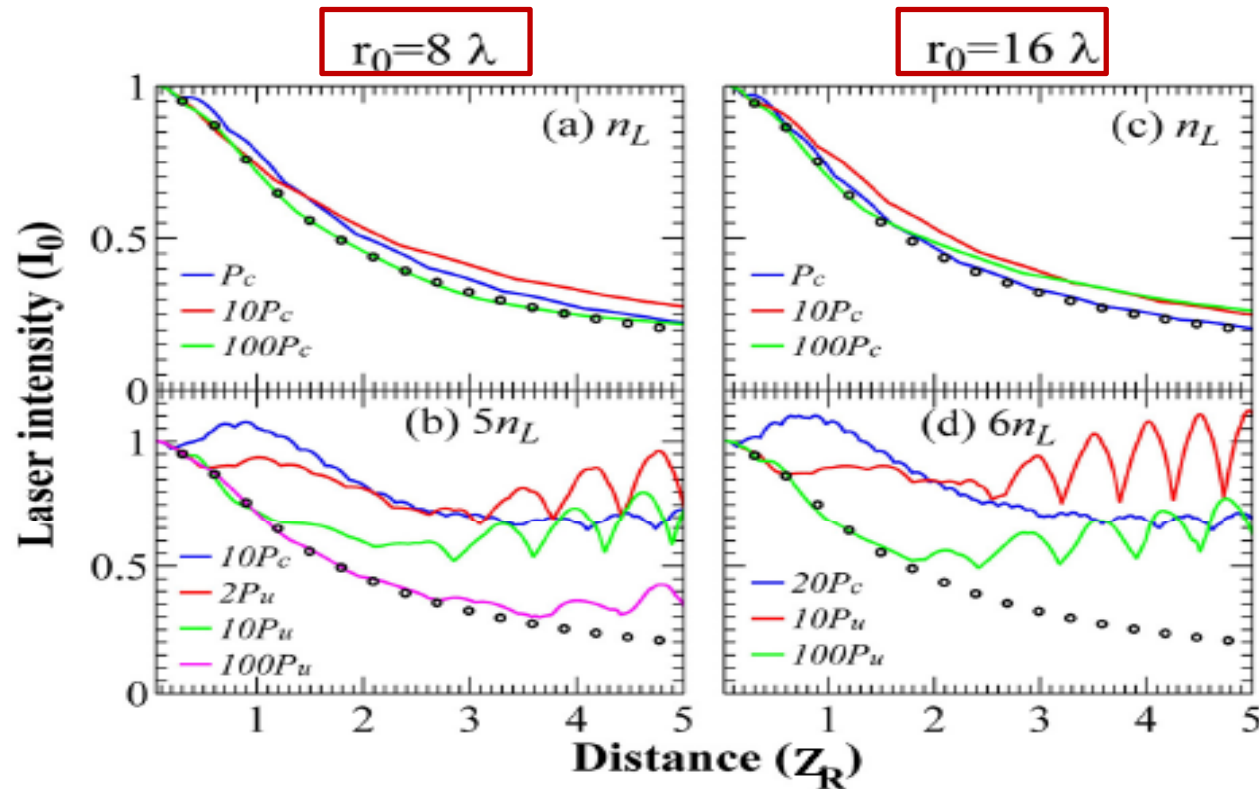


## Verification of $n_L$ and $P_u$ by PIC simulations



- 1)  $n_e \leq n_L$ , self-focusing never occurs with any laser power;  
 $n_e = 4n_L$ , self-focusing starts to appear with  $P_0 = 10P_c$
- 2)  $P_0 = 5P_u$ , ponderomotive-force defocusing starts to appear obviously;  
 increasing  $P_0$ , the laser approaches to the propagation in vacuum
- 3) 2D PIC results are similar ( $4n_L \rightarrow 5n_L$ ,  $5P_u \rightarrow 2P_u$ )

## 2D PIC simulations with larger beam radius $r_0$



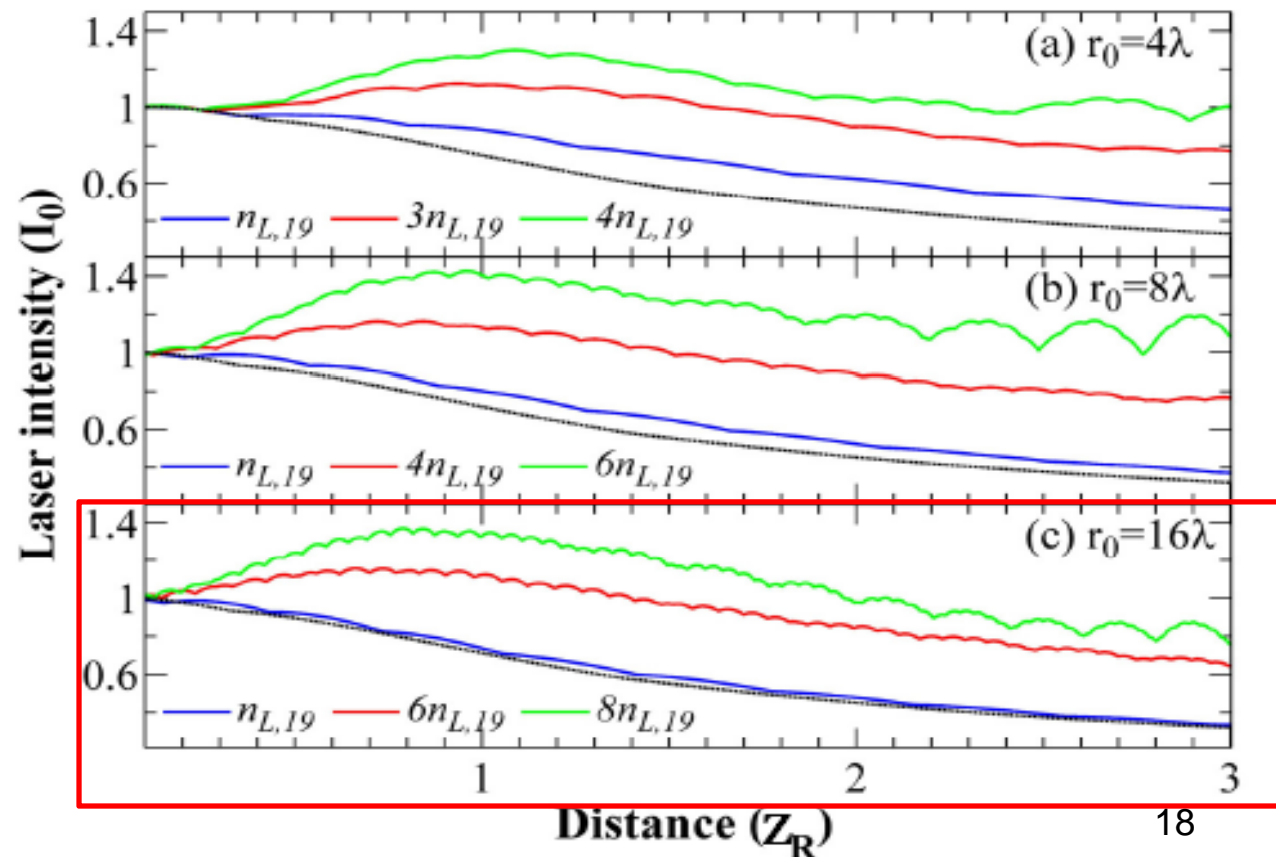
- With  $n_{e0} = 5n_L$  and  $6n_L$ , self-focusing starts to appear for  $r_0 = 8$  and  $16 \mu\text{m}$
- With  $P_0 = 2P_u$  and  $10P_u$ , defocusing starts to appear obviously
- Our theory model agrees with simulation better with smaller  $r_0$

## Further examination with given $I_0$

$$P_c \leq P_0 \leq P_u \Rightarrow \begin{aligned} n_{L,19} &= 0.077n_c(\lambda^2/r_0^2), \quad I_0 = 10^{19} \text{ Wcm}^{-2} \\ n_{L,21} &= 2.5n_c(\lambda^2/r_0^2), \quad I_0 = 10^{21} \text{ Wcm}^{-2} \end{aligned}$$

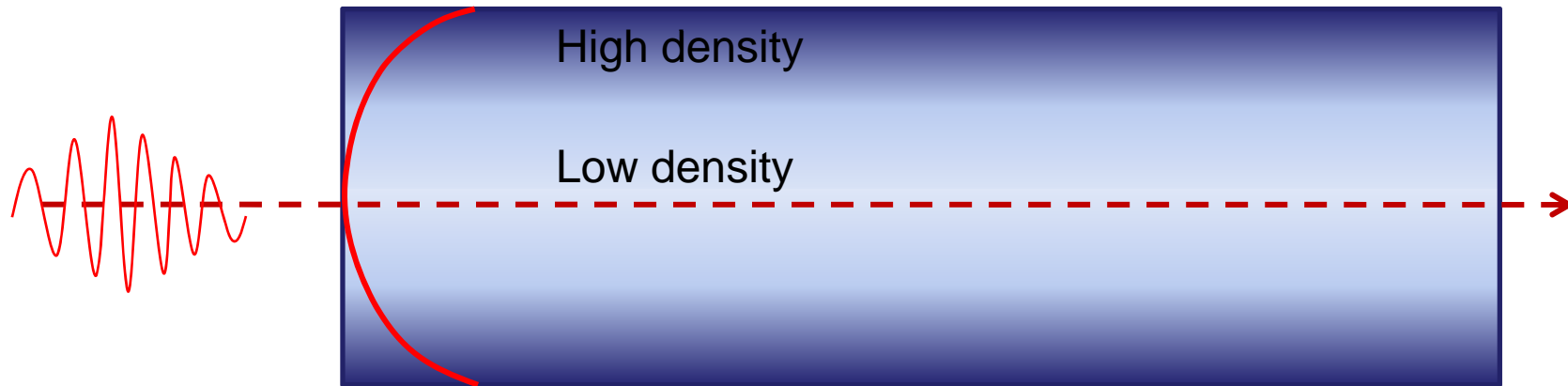
For a given laser intensity, higher densities are favorable for self-guiding.

$$I_0 = 10^{19} \text{ Wcm}^{-2}$$

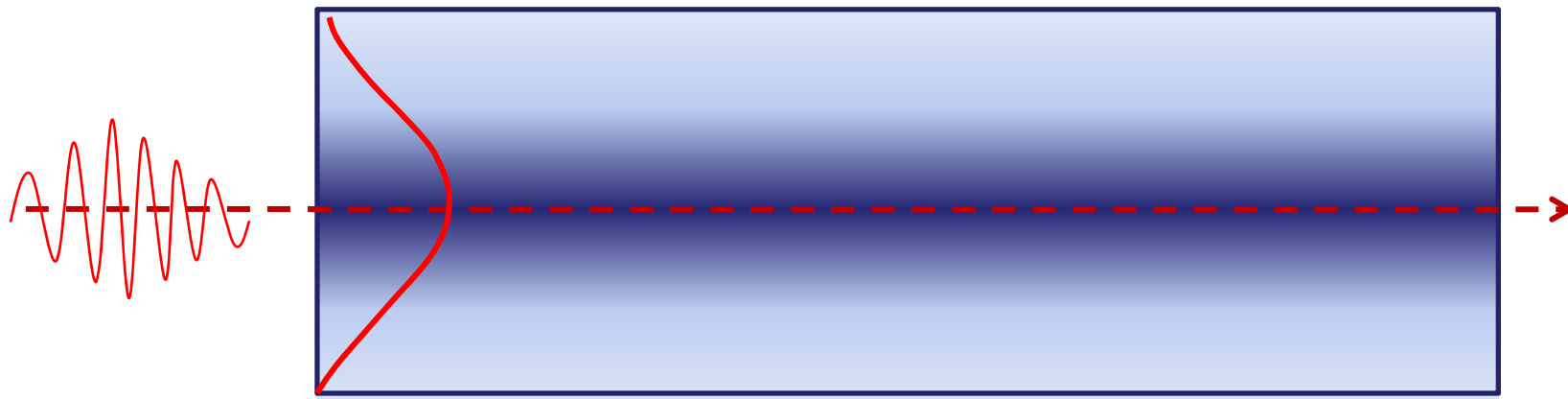


# Effect of inhomogeneous plasma

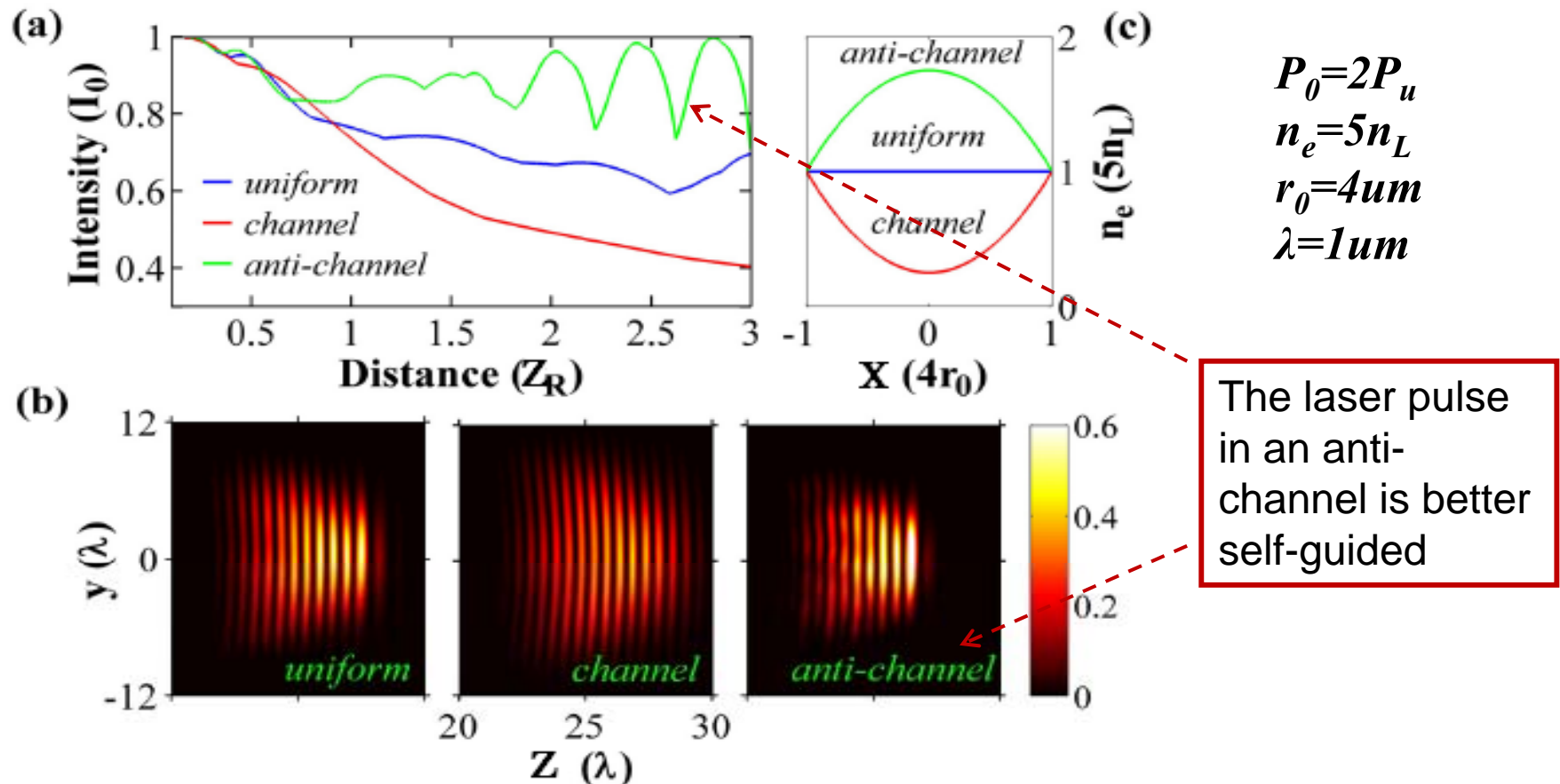
Case 1: normal plasma channel



Case 2: plasma anti-channel



# A plasma anti-channel may be preferred for self-guided propagation at high peak power over 1PW



This agrees with our theory that there is a lower limit of the plasma density for self-guided propagation.

$$P_c < P < P_u \Rightarrow n_{e0} > 0.074 n_c (\lambda^2 / r_0^2)$$

# Summary

- We demonstrate that transverse ponderomotive force may lead to defocusing at high peak laser powers, e.g., PW lasers.
- Power threshold for ponderomotive defocusing or an upper-limit power  $P_u$  for self-focusing is given as a function of initial plasma density  $n_{e0}$  and laser spot size  $r_0$ . In order to have self-guided propagation of laser pulses, the laser power  $P$  should satisfy  $P_c < P < P_u$ .
- A lower-limit density  $n_L$  for self-guiding is given.
- At or above PW with  $P > P_u$ , preformed plasma channels may be not favorable for laser self-guiding. In stead, an anti-channel may be preferred.

**Thank You !**